

Tamm oscillations in semi-infinite nonlinear waveguide arrays

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Abstract

We demonstrate numerically the existence of nonlinear Tamm oscillations at the interface between a substrate and one-dimensional waveguide array with both cubic and saturable, self-focusing and self-defocusing nonlinearity. Light is trapped in the vicinity of the boundary of the array due to the interplay between the repulsive edge potential and Bragg reflection inside the array. In the special case when this potential is linear these oscillations reduce themselves to surface Bloch oscillations.

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The problem of surface waves which could exist at the interface between two different media has been studied for several decades. These waves are, for example, investigated at a metal-vacuum interface [1], air-water interface [2], at the interface between two dielectrics [3], and in finite anti-dot lattices [4]. Tamm was the first to consider neglected edge effects in a semi-infinite Kronig-Penney model [5]. He has discovered that strongly localized surface states could appear providing that the surface potential perturbation is strong enough. However, due to various experimental difficulties, his discovery was awaiting more than half a century on the experimental confirmation in semiconductor superlattices [6]. From the practical point of view, surface waves represent convenient tools to investigate the properties of material interfaces. Furthermore, they could be implemented in various devices such as bio-sensors [7] and polariton lasers [8].

Recently it has been suggested that these surface states may also exist at the interface between a homogenous medium (substrate) and a nonlinear waveguide array (NWA) [9]. NWA represent arrays of evanescently coupled optical waveguides. Up to date, NWA have been successively fabricated in materials exhibiting cubic [10, 11, 12], quadratic [13], saturable [14, 15], and nonlocal nonlinearity [16]. Discrete solitons [10, 13, 14, 17], breathers [18], diffraction management [11], modulational instability [12, 15, 16], and Bloch oscillations [19] are just a few examples of phenomena which have been observed in such systems.

Very recently, the first experimental observation of discrete surface solitons in AlGaAs NWA exhibiting a cubic self-focusing nonlinearity [20] has triggered further investigations of surface waves at the interface between a NWA and a substrate. The existence of surface gap solitons in the lattice with cubic self-defocusing nonlinearity has been reported in Ref. [21]. A crossover from nonlinear surface states to discrete solitons was studied, too [22]. In these two papers it has been revealed that the vicinity of the edge enables a stable propagation of various localized modes, such as flat-top modes and inter-site ones (mode B) [23]. Very recently, strongly localized surface waves have been independently observed in NWA exhibiting saturable [24, 25] and quadratic nonlinearity [26], respectively. The fact that a surface can support the existence of localized states has been, for example, further exploited in vector [27], nonlocal [28] as well as binary NWA [29], and in a soliton array case [30].

In this Letter we reveal the existence of Tamm oscillations at the edge of a semi-infinite NWA. These oscillations are the result of an interplay between a repulsive potential which

originates from the boundary of the array and the array's periodicity. We calculate this potential for a few different media and reveal that Tamm oscillations are more likely to occur in systems with stronger coupling. In a special case when the repulsive potential is a linear function of the distance from the edge of the NWA Tamm oscillations reduce to the well-known Bloch oscillations [31, 32, 33], which have considerable potential in all-optical switching, amplification, and steering [19, 32].

The propagation of light in periodically stratified structures such as NWA can be described analytically either by the Floquet-Bloch model [15, 34] or by a coupled-wave theory [9, 10, 13, 17]. The model equation within the first approach reads:

$$i\frac{\partial E}{\partial y} + \frac{1}{2k}\frac{\partial^2 E}{\partial z^2} + k\frac{n(z) + \Delta n_{nl}}{n_s}E = 0. \quad (1)$$

The propagation coordinate is along the y -axis, the amplitude of the electrical field is denoted by E , while $k = 2\pi n_s/\lambda$ represents the wave number. Here, λ is the wavelength of the used light in vacuum, and n_s is the extraordinary refractive index of the substrate. The periodical modulation of the refractive index which defines the nonlinear WA is denoted by $n(z)$ while Δn_{nl} is the nonlinear refractive index change ($\Delta n_{nl} \ll n_s$). In media with cubic nonlinearity $\Delta n_{nl} = \Delta n_0 I$ while in media with saturable nonlinearity we have $\Delta n_{nl} = \Delta n_0 I/(I + I_d)$, where I is the peak light intensity and I_d is the so-called dark irradiance [14].

On the other hand, light propagation through NWA may be described, within the tight-binding approximation, by the following set of conveniently normalized, linearly coupled, nonlinear, ordinary differential equations:

$$i\frac{dU_n}{d\xi} + C(U_{n+1} + U_{n-1} - 2U_n) - g(|U_n|^2)U_n = 0, \quad (2)$$

where $g(|U_n|^2) = \beta|U_n|^2$ in cubic (c) media and $g(|U_n|^2) = \beta|U_n|^2/(1 + |U_n|^2)$ in saturable (s) media [35], while the nonlinearity coefficient $\beta < 0$ for the self-focusing (f) and $\beta > 0$ for the self-defocusing (d) case, respectively. Here C is the coupling constant, ξ is the propagation coordinate and U_n is the normalized electric field envelope in the n -th waveguide [36]. Integral of motions are power and Hamiltonian:

$$\begin{aligned} P &= \sum_n |U_n|^2, \\ H_s &= \sum_n \{C|U_{n-1} - U_n|^2 - \beta[\ln(1 + |U_n|^2) - |U_n|^2]\}, \\ H_c &= \sum_n [C|U_{n-1} - U_n|^2 + \frac{\beta|U_n|^4}{2}]. \end{aligned} \quad (3)$$

Assuming stationary solutions of staggered form $U_n = F_n \exp[i(-\nu\xi + n\pi)]$ (ν represents soliton frequency) for defocusing cases, and of unstaggered form $U_n = F_n \exp(-i\nu\xi)$ for focusing cases, together with the assumption $|F_0| \gg |F_{\pm 1}| \gg |F_{\pm 2}|$ for on-site (A) mode and $|F_{\pm 1}| \gg |F_{\pm 2}| \gg |F_{\pm 3}|$ for inter-site (B) mode, one may find the following expressions for the maximal amplitude in the array:

$$F_{0s(f,d)} = \sqrt{\frac{\nu - 2C}{\beta - \nu + 2C}}, \quad F_{0c(f,d)} = \sqrt{\frac{\nu - 2C}{\beta}}, \quad (4)$$

$$F_{1s(f)} = \sqrt{\frac{\nu - C}{\beta - \nu + C}}, \quad F_{1s(d)} = \sqrt{\frac{\nu - 3C}{\beta - \nu + 3C}}, \quad (5)$$

$$F_{1c(f)} = \sqrt{\frac{\nu - 3C}{\beta}}, \quad F_{1c(d)} = \sqrt{\frac{\nu - C}{\beta}}. \quad (6)$$

In addition for A mode $F_{n(>0)} = \alpha^n F_0$ and for B mode $F_{n(>1)} = \alpha^{n-1} F_1$, in both saturable and cubic media. Here $\alpha = \pm C/(\nu - 2C)$ for (d) and (f) case, respectively. Examples of the oscillatory behavior of both on-site mode (mode A) and mode B are depicted in Fig. 1. Here the energy is too low to overcome the repulsive potential from the edge of the array so both modes start to move away from the interface until they are back-reflected because of the Bragg condition. As this traversing is usually accompanied with radiation reflected modes loose power and eventually do not reach back to the first channel of the array. As a result, consequent oscillations have longer and longer periods and modes gradually run away from the edge. The trapping of localized modes at one channel after one or more oscillations is possible as well. Here is interesting to mention that surface waves could exist due to the balance between self-bending and deflection from the edge of a bulk photorefractive crystal, too [37]. Equivalently, the behavior of moving localized modes can be interpreted by the interplay between a repulsive force (due to the boundary effects) and the effective Peierls-Nabarro potential (due to system discreteness) [35].

We calculate the effective repulsive potential as the difference between the truncated potential and a reference potential with periodic boundary condition, and present results for symmetric localized structures centered on-site in an array that consists of 13 elements. As the peak of this localized structure (with the highest amplitude F_0) approaches the edge more and more terms of the corresponding Hamiltonian have to be truncated. For example, if the peak of mode A resides in the 6th channel one can obtain the following expressions

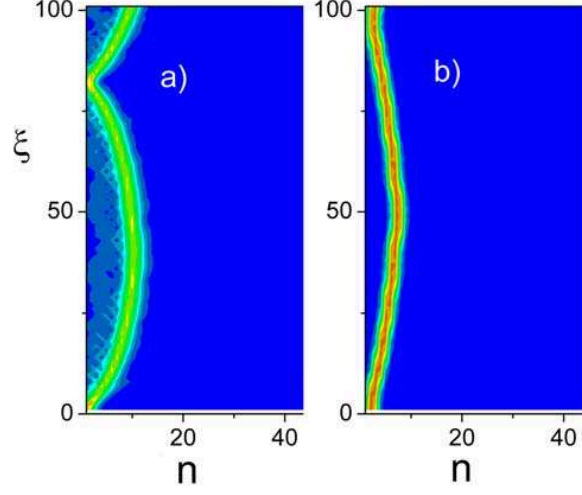


FIG. 1: Tamm oscillations of a) mode A with $C = 0.5$, $|\beta| = 3.34$, and soliton frequency $\nu = 2.3$ launched into the first channel of the array, and b) mode B with $C = 0.5$, $|\beta| = 3.34$, and soliton frequency $\nu = 4.83$ launched into the second and third channel of the array.

for the repulsive potential:

$$\begin{aligned}
 V_{rep.c}^{(1)} &= C|F_0|^2\alpha^{10} - C|F_0|^2(1 + \alpha)^2\alpha^{10} - 0.5\beta|F_0|^4\alpha^{24}, \\
 V_{rep.s}^{(1)} &= C|F_0|^2\alpha^{10} - C|F_0|^2(1 + \alpha)^2\alpha^{10} \\
 &\quad - \beta|F_0|^2\alpha^{12} + \beta \ln(1 + \alpha^{12}|F_0|^2),
 \end{aligned} \tag{7}$$

in both saturable and cubic media in defocusing and focusing cases. After a straightforward calculation it is possible to get explicit expressions for $V_{rep}^{(2)}$, $V_{rep}^{(3)}$, etc. where the number in the superscript denotes the number of the truncated channels. For example, for the Tamm state in the self-focusing cubic case we have found:

$$\begin{aligned}
 V_{rep.cf}^{(6)} &= C|F_0|^2 \\
 &\quad - C|F_0|^2(1 + \alpha)^2(\alpha^{10} + \alpha^8 + \alpha^6 + \alpha^4 + \alpha^2 + 1) \\
 &\quad - 0.5\beta|F_0|^4(\alpha^{24} + \alpha^{20} + \alpha^{16} + \alpha^{12} + \alpha^8 + \alpha^4).
 \end{aligned} \tag{8}$$

Note that here $\beta < 0$ and $\alpha = -C/(\nu - 2C)$. The dependence of the repulsive potential on the distance from the edge of the array is presented in Fig. 2. Stronger coupling (i.e. shorter coupling length) results in stronger repulsion (see Fig. 2a) while stronger nonlinearity decreases the repulsive potential (Fig. 2b). Beams which are strongly pushed off from the edge will experience Bragg reflection inside the array earlier than weakly rejected beams.

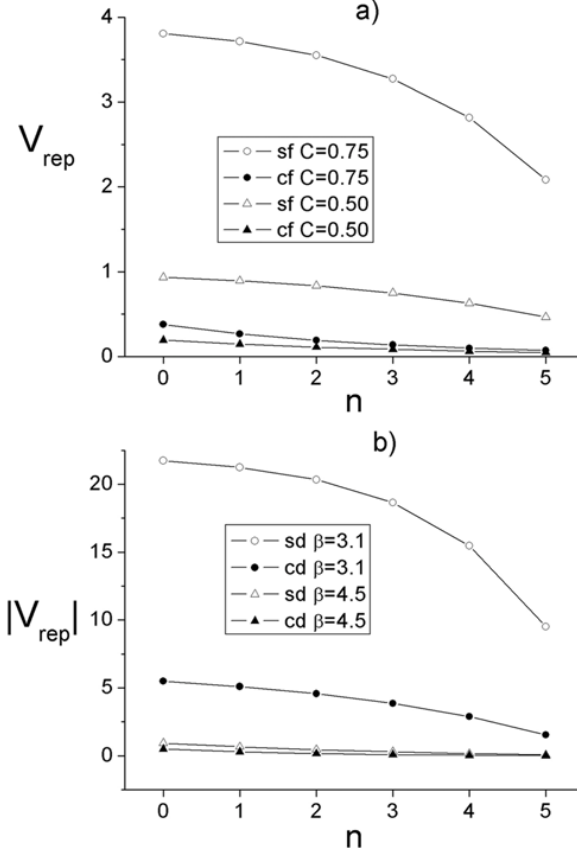


FIG. 2: Dependence of the effective repulsive potential on the distance from the interface between a substrate and the array. a) Self-focusing case: $|\beta| = 3.1$, $|F_0|^2 = 0.24$ and different values of C . b) Self-defocusing case: $C = 0.5$, $|F_0|^2 = 0.172$, and different values of β .

Thus, the same input beam will have shorter spatial periods of Tamm oscillations in arrays with stronger coupling and weaker nonlinearity.

The value of the corresponding repulsive potentials for both cubic and saturable, self-focusing and self-defocusing cases also depends on the soliton frequency ν . For a fixed value of this frequency and fixed values of parameters C and β , the following relation is usually fulfilled: $V_{rep}^{sd} > V_{rep}^{cd} > V_{rep}^{sf} > V_{rep}^{cf}$ but interior elements may permute their position as well. This relation may explain why this effect has not been observed in recent studies in AlGaAs waveguide arrays exhibiting a cubic self-focusing nonlinearity [9, 20]. Interestingly, in the case when V_{rep} is a linear function from the interface distance the input beam will experience Bloch oscillations [19], which have a considerable potential for application in all-optical devices as reported, for example, in Ref. 32. Restricting ourselves here only

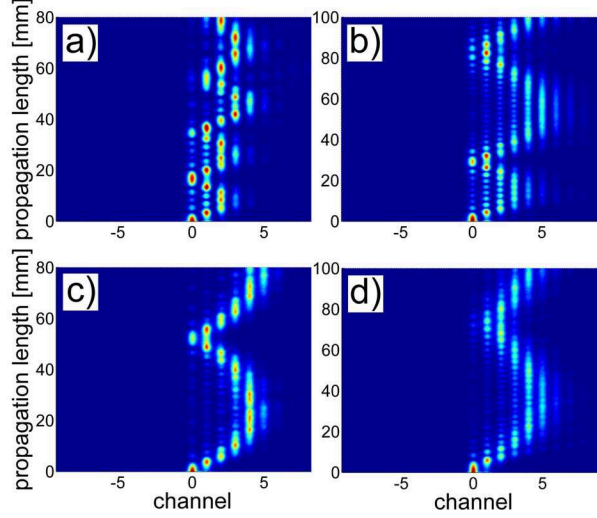


FIG. 3: (Color online) Tamm oscillations in a waveguide array where channels are $4\,\mu\text{m}$ wide and separated by $4.4\,\mu\text{m}$ while $\Delta n_0 = 3.7 \times 10^{-4}$. The input pattern with amplitude ratios of $1 : 0.5 : 0.1$ has been launched into the first channel. a) Self-defocusing saturable case $\Delta n_{nl} = 6.2 \times 10^{-4}$, $r = I/I_d = 6$, b) self-defocusing cubic case $\Delta n_{nl} = 4.42 \times 10^{-4}$, c) self-focusing saturable case $\Delta n_{nl} = 3.34 \times 10^{-4}$, $r = 6$, and d) self-focusing cubic case $\Delta n_{nl} = 2.62 \times 10^{-4}$.

to NWA, this phenomenon was studied in curved NWA [33], and in NWA with a linearly growing effective refractive index [19]. Please note that, from the surface's point of view, Tamm oscillations do not belong to nonlinear effects (i.e. they exist for light intensities lower than the threshold for the onset of highly nonlinear Tamm states [9, 20, 21, 22, 24, 25, 26]).

In order to check our findings we performed additional simulations based on a numerical solution of Eq. (1). We, arbitrarily, have used the parameters which are achievable in lithium niobate waveguide arrays exhibiting a self-defocusing saturable nonlinearity ($\Delta n_0 = 3.7 \times 10^{-4}$, $\Delta n_{nl} \leq 0.001$) and green light with $\lambda = 532\,\text{nm}$. For such samples the periodically modulated refractive index can be well approximated by a \cos^2 function [15]. Numerical results which have been obtained by virtue of a beam propagation method are given in Fig. 3. Note that in reality this effect occurs in a highly elongated space ($20\,\text{mm} \times 0.1\,\text{mm}$ in Fig. 3a).

As the position of the input beam shifts towards the interior of the array, the period of Tamm oscillations increases. Fig. 4 may be understood as a numerical proof that these oscillations are indeed a linear surface effect. Namely, the light intensity necessary to form Tamm oscillations (Fig. 4a) is not high enough to form a surface soliton [9, 20, 21, 22, 24,

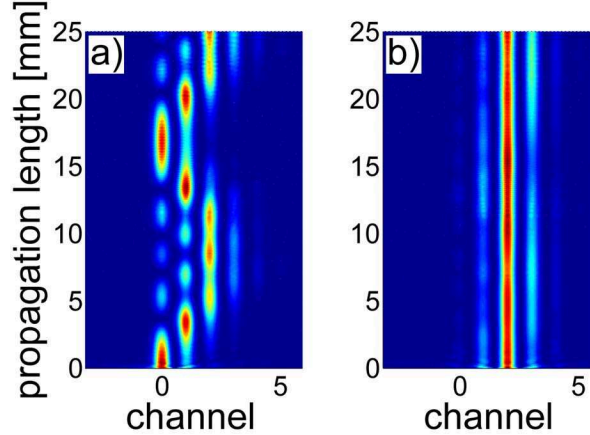


FIG. 4: (Color online) Light propagation in a saturable nonlinear NWA with the same input profile and lattice parameters as in Fig. 3. Here $\Delta n_{nl} = 6.21 \times 10^{-4}$ and $r = 6$. The input beam is launched into a) the first channel and b) into the third channel of a semi-infinite lattice.

25, 26] but suffices to form a narrow breather in a channel which is only two channels away from the substrate-array interface (Fig. 4b).

Experimentally, Tamm oscillations may be observed, for example, in one-dimensional lithium niobate waveguide arrays, where sample lengths up to 50 mm have been achieved [24, 26]. However, up to now we failed in a direct observation of Tamm oscillations from the top of the sample because of the rather low average level of scattered light that is superimposed by larger scattering amplitudes from small surface defects. An alternative approach has been outlined in Ref. [19], and corresponding experiments may be performed in the near future.

In conclusion, we have demonstrated numerically the existence of Tamm oscillations at the interface between a substrate and a one-dimensional homogeneous nonlinear waveguide array. Light is trapped in the vicinity of the edge of the array due to the interplay between the edge repulsion and Bragg reflection. Approximate analytical expressions for the repulsive truncated potential are given for different types of nonlinear interactions as well as for different system parameters. These oscillations reduce to the case of Bloch oscillations when the repulsive potential is a linearly decreasing function of the distance from the edge of the semi-infinite nonlinear waveguide array.

Acknowledgments

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- [1] A. Otto, Zeit. Phys. **216**, 398 (1968).
- [2] D. Middleton and R. H. Mellen, J. Acoust. Soc. Am. **90**, 741 (1991).
- [3] W. J. Tomlinson, Opt. Lett. **5**, 323 (1980).
- [4] P. H. Rivera et al., Phys. Rev. B **64**, 035313 (2001).
- [5] I. Tamm, Phys. Z. Sowjetunion **1**, 733 (1932).
- [6] H. Ohno et al., Phys. Rev. Lett. **64**, 2555 (1990).
- [7] C. Chou et al., Opt. Exp. **14**, 4307 (2006).
- [8] A. Kavokin, I. Shelykh, and G. Malpuech, Appl. Phys. Lett. **87**, 261105 (2005).
- [9] K. G. Makris et al., Opt. Lett. **30**, 2466 (2005).
- [10] H. Eisenberg et al., Phys. Rev. Lett. **81**, 3383 (1998).
- [11] H. Eisenberg et al., Phys. Rev. Lett. **85**, 1863 (2000).
- [12] J. Meier et al., Phys. Rev. Lett. **92**, 163902 (2004).
- [13] R. Iwanow et al., Phys. Rev. Lett. **93**, 113902 (2004).
- [14] N. K. Efremidis et al., Phys. Rev. E **66**, 046602 (2002).
- [15] M. Stepić et al., Opt. Lett. **31**, 247 (2006).
- [16] M. Peccianti et al., Nature **432**, 733 (2004).
- [17] D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 794 (1988).
- [18] S. Flach and C. R. Willis, Phys. Rept. **295**, 181 (1998).
- [19] R. Morandotti et al., Phys. Rev. Lett. **83**, 4756 (1999).
- [20] S. Suntsov et al., Phys. Rev. Lett. **96**, 063901 (2006).
- [21] Y. V. Kartashov, V. A. Vysloukh, and L. Torner, Phys. Rev. Lett. **96**, 073901 (2006).
- [22] M. I. Molina, R. A. Vicencio, and Yu. S. Kivshar, Opt. Lett. **31**, 1693 (2006).
- [23] F. Lederer, S. Darmanyan and A. Kobayakov, Discrete solitons, in *Spatial solitons*, ed. by S. Trillo and W. Torruellas (Springer, Berlin 2001), p. 269.

- [24] E. Smirnov et al., Opt. Lett. **31**, 2338 (2006).
- [25] C. Rosberg et al., arXiv:nlin.PS/0603202 (2006).
- [26] G. A. Siviloglou et al., Opt. Exp. **14**, 5508 (2006).
- [27] J. Hudock et al., Opt. Exp. **13**, 7720 (2005).
- [28] Y. V. Kartashov et al., arXiv:nlin.PS/0606160 (2006).
- [29] M. I. Molina et al., Opt. Lett. **31**, 2332 (2006).
- [30] Y. V. Kartashov et al., Opt. Lett. **31**, 2329 (2006).
- [31] F. Bloch, Z. Phys. **52**, 555 (1928).
- [32] U. Peschel, T. Pertsch, and F. Lederer, Opt. Lett. **23**, 1701 (1998).
- [33] N. Chiodo et al., Opt. Lett. **31**, 1651 (2006).
- [34] P. Yeh, A. Yariv and C.-S. Hong, J. Opt. Soc. Am. **67**, 423 (1977).
- [35] Lj. Hadzievski et al., Phys. Rev. Lett. **93**, 033901 (2004).
- [36] M. Stepić et al., to be published in Opt. Commun. (2006).
- [37] M. Cronin-Golomb. Opt. Lett. **20**, 2075 (1995).